Letters

Accuracy of Equations for Coupled Slab Lines

P. W. VAN DER WALT

The determination of the characteristic impedance of coupled slab lines has been a subject of investigation for many years. Following on Frankel's pioneering work [1], Chisholm obtained an accurate analytical solution for the odd-mode impedance of a pair of coupled slab lines [2]. In 1980, Levy presented Chisholm's equations, together with those due to McDermott for the even-mode impedance, in a general form suitable for calculation [3]. He also confirmed the accuracy of the equations with an efficient numerical procedure.

Stracca *et al.* recently published a new set of equations for the characteristic impedances of coupled slab lines [4]. These were found by adding correcting terms to the approximate equations of Frankel [1] and Honey [5] to improve their agreement with a set of accurate numerical results.

In the design of interdigital filters, the capacitance matrix is of prime importance [6]. This matrix is directly related to the normalized capacitances C_g/ϵ between bars and earth planes, and C_m/ϵ between adjacent bars, and can be calculated from even- and odd-mode impedance data [3]. The normalized capacitance C_m/ϵ is a sensitive indicator of accuracy as it is proportional to the often small difference between the odd- and even-mode capacitances.

The results contained in [3] and [4] are compared in Table I for $d/b = 0.400$ in terms of C_g/ϵ and C_m/ϵ. The parameter s/b denotes the spacing between bars at their closest points. For these calculations, equation (4) in [4] was corrected by replacing the first subtraction with an addition.

With Chisholm and McDermott's results taken as the basis of comparison, the percentage differences between the various results are shown in Figs. 1 and 2. The numerical results presented by Stracca *et al.* [4] are in excellent agreement with those of Chisholm and McDermott, and are also in good agreement with the numerical results obtained by Levy. In contrast to this, however, their new equations show relatively poor agreement with the numerical results. A comparison of results for other values of d/b confirms the conclusion that Chisholm and McDermott's equations are the preferred set for filter design.

TABLE I
NORMALIZED CAPACITANCE

	C&McD (Eqns)		Levy (Num)		Stracca (Num)		Stracca (Eqns)	
s/b	C_g/ϵ	C_m/ϵ	C_g/ϵ	C_m/ϵ	C_g/ϵ	C_m/ϵ	C_g/ϵ	C_m/ϵ
0.080	2.891	3.456	2.874	3.567	-	-	2.936	3.324
0.100	2.991	2.968	-		2.995	3.012	3.051	2.918
0.120	3.088	2.599	3.088	2.635	-	-	3.141	2.568
0.160	3.278	2.068	3.266	2.095	-	-	3.288	2.032
0.200	3.459	1.700	3.451	1.710	3.462	1.705	3.424	1.660
0.240	3.630	1.426	3.645	1.431	-	-	3.563	1.391
0.300	3.870	1.122	-	-	3.870	1.124	3.778	1.102
0.400	4.217	0.781	4.216	0.783	4.219	0.782	4.130	0.779
0.600	4.727	0.402	4.727	0.401	4.728	0.402	4.683	0.409
0.760	4.989	0.241	4.989	0.241	-	-	4.966	0.246
0.800	5.039	0.212	-	-	5.040	0.212	5.020	0.217
1.000	5.219	0.113	-	-	5.219	0.113	5.210	0.116

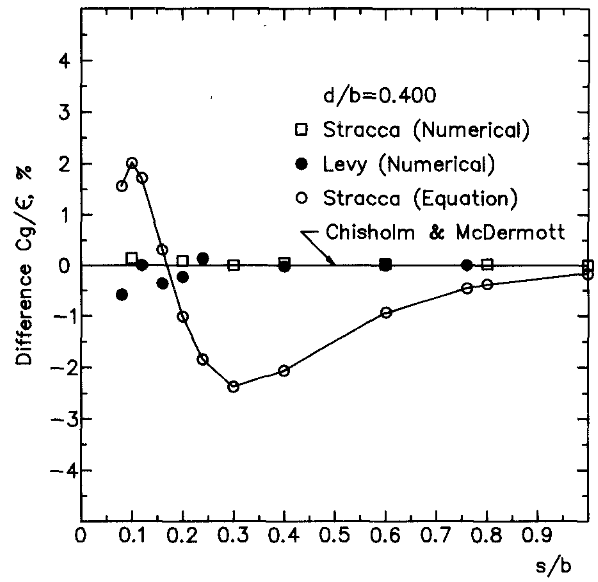

Fig. 1. Differences between values of C_g/ϵ. Chisholm and McDermott's equations are used as the basis of comparison.

Reply[1] *by G. B. Stracca, G. Macchiarella, and M. Politi*[2]

The letter of Prof. van der Walt requires some comments.

a) Our paper [4] is mainly concerned with the numerical analysis of various configurations of slab lines. Equations (5) and (6), obtained through interpolations of the numerical computed data and clearly presented as approximated formulas, have been included in the paper as a secondary result.

The interest of these formulas, however, is their very simple expressions, obtained by adding correcting terms into Frankel's approximated expression for the slab line characteristic impedance. Moreover, they can be easily implemented even with programmable calculators and are far simpler than Chisholm's equations in the form presented by Levy [3].

b) Our formulas have been derived for computing even and odd characteristic impedances (Z_{ce} and Z_{co}) of coupled slab lines; it is not correct to refer to other parameters, such as C_g/ϵ and C_m/ϵ (as done in van der Walt's letter) in order to evaluate the accuracy. In fact, small errors in Z_{ce} and Z_{co} may be reflected in greater errors in derived parameters, especially when equations are obtained from interpolations of numerical data.

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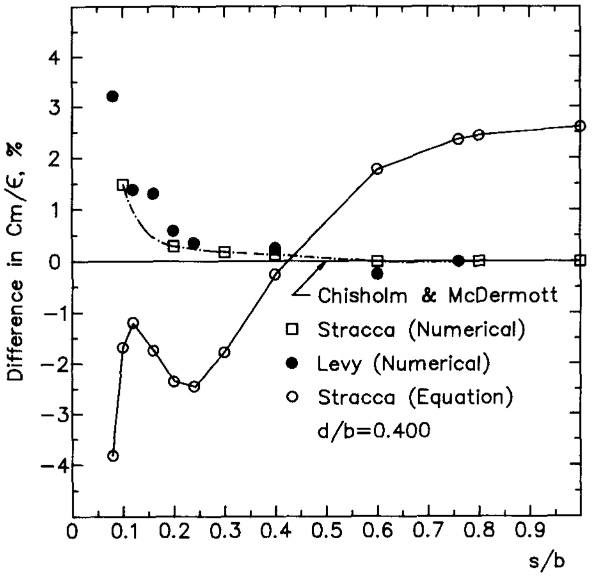

Fig. 2. Differences between values of C_m/ϵ. Chisholm and McDermott's equations are used as the basis of comparison. The broken line is a fit to values of C_m/ϵ derived from the accurate numerical results in [4].

Deviations between numerical data and values obtained with formulas (5) and (6) of [4] are much smaller than those in C_g/ϵ and C_m/ϵ as reported in the above letter. For example, for $d/b = 0.4$ (the same utilized here in Figs. 1 and 2) in the range of validity of our interpolating expressions ($0.12 \leqslant s/b \leqslant 3.6$) [4], the maximum deviations between numerical and interpolated values of Z_{ce} and Z_{co} are

$$0\% \leqslant \Delta Z_{ce} \leqslant 1.0\%$$
$$-0.03\% \leqslant \Delta Z_{co} \leqslant 1.4\%.$$

c) In the design of coupled-line filters or directional couplers, some parameters derived from Z_{ce} and Z_{co} are of importance, such as

$$Z_m = (Z_{ce} + Z_{co})/2$$
$$Z_g = \sqrt{Z_{ce} * Z_{co}}$$
$$m = (Z_{ce} - Z_{co})/(Z_{ce} + Z_{co}).$$

As far as the accuracy is concerned in evaluating these parameters, it is more convenient to use "ad hoc" expressions (obtained from interpolations of numerical data) than an indirect computation through the expressions for Z_{ce} and Z_{co}. A set of such equations, useful for design problems, is reported in [7]. For instance, the parameter m, which directly governs coupling both in directional couplers and in interdigital and combline filters, can be expressed as

$$m = M(d/b, s/b) * (\ln(\coth(\pi c/2b)))/(\ln(\coth(\pi d/4b)))$$

where $M(d/b, s/b)$ is a correcting term whose expression is reported in [7]. The above equation guarantees an accuracy with respect to the numerical data of better than 1 percent for $d/b = 0.4$.

Finally, we take this opportunity to correct a typing error that occurred in formula (4) of [4]: the exponent of the first term in square brackets should be $-1/2$.

References

[1] S Frankel, "Characteristic impedance of parallel wires in rectangular troughs," *Proc. IRE*, vol 30, pp. 182–190, Apr. 1942.
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Corrections to "New Quasi-Static Models for the Computer-Aided Design of Suspended and Inverted Microstrip Lines"

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The following errors appeared in the equations of the above paper[1]:

a) In equation (3), c_i should be read as C_i.

b) In equation (A2g), d^3 should be replaced by f^3.

c) In equation (A3h), (486.7425 + 7425 + 279.8...) should read (486.7425 + 279.8...), i.e., delete 7425.

d) In equation (A3k), −919.36661 should read +919.3661.

e) In equation (A2p), 161.2689 f^2 should read 161.2689 f^3.

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[1] R. S. Tomar and P. Bhartia, *IEEE Trans Microwave Theory Tech*, vol. MTT-35, pp. 453–457, Apr 1987.